

The Collatz Conjecture as Irreversible Sedimentation in the Inversion Topology

Rail confinement, 3:1 compression, the attractor (1, 8), and why no finite
archive closes

Filipe Bastos

Architecture of Relational Coherence (ARC), D2technologie Inc., Canada

Version 2.2, 24 October 2026 (corrects version 2.1, March 2026)

Contents

Abstract	2
0. What changed from version 2.1	2
1. Introduction	3
2. Preliminaries: the ARC digit system	4
3. Theorem 1: compression onto the right rail	4
4. Theorem 2: rail alternation	4
5. Theorem 3: absorption of the central axis	5
6. Theorem 4: the attractor (1, 8)	5
7. The convergence question, restated	5
8. The bridge from address to magnitude	6
9. Theorem 5: halving and the powers of 2	6
10. Theorem 6: the digital-root channel	6
11. Theorem 7: twin prime products	6
12. What replaces Theorem 8	7
13. Worked examples	9
14. Status and falsifiability	10
15. Conclusion	10
References	11

To Sophie, who asks every morning: "Did your numbers come home last night?"

To Soleil, Delfine, Justin and Yanie: four children, four inversion pairs. Every night you scatter into chaos. Every morning you converge to 1. You are my living Collatz proof.

To my parents, Joaquim and Agostinha, who taught me that all roads lead home.

Abstract

The Collatz conjecture (1937) asserts that iterating $n \mapsto n/2$ for even n and $n \mapsto 3n + 1$ for odd n always reaches 1. This paper reads the map through the Architecture of Relational Coherence (ARC), on the digit alphabet $\{1, \dots, 9\}$ with the inversion $\varphi(x) = 9 - x$.

Version 2.1 (March 2026) stated eight theorems and claimed, in its Theorem 8, to reduce the conjecture to “a finite ergodic question” on 18 residue classes. **Theorem 8 is retracted** (8 July 2026): it was computed on the smallest representative of each class, and two integers, 37 and 31, refute it. Version 2.2 keeps every exact result of 2.1, removes nominatively what was false, and puts in its place what the ARC corpus has proved since (Volume 4, *La Théorie des nombres*, Part III).

What is proved: Theorems 1 to 5 and 7 of version 2.1 (the 3:1 compression onto the right rail, the period-6 rail alternation, the absorption of the central axis, the terminal cycle at (1, 8), the coupling of halving with the powers of 2, the twin product theorem), properties (a) to (d) of Theorem 6, and the new results: the quarter dichotomy, the unbounded growth chains, the strangeness of the two bricks 3 and 2, the +1 as the returned half, the exact balance of an orbit, the gate 8, the finite-time release of every run of 1s, the seed below zero of every repeated rising word, and a corollary: **no finite archive of residue-class certificates can close the conjecture on its own.**

What is not proved: the conjecture. It stays open, as the work site P-COL-1, with an explicit criterion for any candidate proof.

Statuses. Every statement carries the ARC corpus status: **Th** (theorem: proof, or finite computation verified), **F** (received fact: what science holds in 2026 by its own method), **L** (reading, declared as such), **P** (prediction), **Chantier** (open work site). Every computation is redone by the script `collatz_v22_verif.py`, attached to this paper (35 checks).

0. What changed from version 2.1

The rule followed is that of the corpus: *no line removed without a line proposed in its place*, and every retraction kept with its date and its replacement.

Version 2.1	Status in 2.2	Replaced by
Theorem 8, the contraction by classes of 36: table of fixed valuations, “maximum growth chain 3”, two-state chain 5/8 against 3/8, factor 0.662, “finite ergodic question”	Retracted, 8 July 2026	Theorem 21 (quarter dichotomy), Theorem 22 (chains), §12
§ 7.1, “ $3^k \neq 2^m$ excludes cycles”	Retracted, 27 September 2026	Theorem 25: a cycle needs $2^m = 3^k \Pi(1 + 1/(3n_i))$ exactly; exclusion open
§ 7.3, information exhausted in about $\log n$ odd steps, constant 6.95	Retracted, 27 September 2026	Theorem 23 (the root carries no bit of v_2); $L = k + m + 1$
§ 7.2 and § 8, “Section 12 proves it is impossible”	Replaced, 27 September 2026	Theorem 21 and P-COL-1
§ 1.2 “convergence is structurally inevitable”; § 15 “contraction is structural, growth is transient”	No longer stated as proved	The author’s sentences are kept as such; their content is P-COL-1
Theorem 6, uniqueness of the digital-root channel	Downgraded to L	(a) to (d) remain Th
Criterion F5	Reached, hence void	The retraction itself

1. Introduction

1.1 (F) The conjecture. Define $C(n) = n/2$ if n is even and $C(n) = 3n + 1$ if n is odd. The conjecture states that for every positive integer n the iterates reach 1 and enter the cycle $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$. All starting values up to 695×2^{60} have been checked (Barina). Almost all orbits, in the sense of logarithmic density, attain almost bounded values (Tao, 2019; *Forum of Mathematics, Pi*, 2022), which does not say all. Erdős said that mathematics is “not yet ready for such problems”.

1.2 (L) The ARC approach. The Collatz map is read as a sedimentation in the inversion topology of $\{1, \dots, 9\}$. The step $3n + 1$ is a 3:1 compression in digital-root space; halving is a bijection there. Version 2.1 concluded from this that convergence is “structurally inevitable”. That conclusion is not a theorem: the compression acts on the *address* of a number (its digital root), while the descent is governed by the *count of factors 2* (its 2-adic valuation), and Theorem 23 below proves that the first carries no information on the second. The bridge from address to magnitude is exactly what remains open.

1.3 Structure. Sections 2 to 6 and 9 to 11 keep the address-level theorems of version 2.1. Sections 7 and 8 are rewritten. Section 12 replaces the retracted Theorem 8 by what is proved. Section 13 gives worked examples, Section 14 the status, Section 15 the conclusion.

2. Preliminaries: the ARC digit system

2.1 The four inversion pairs. On $\{1, \dots, 9\}$ the inversion $\varphi(x) = 9 - x$ produces four pairs, exhaustive: (1, 8) of span 7, (2, 7) of span 5, (3, 6) of span 3, (4, 5) of span 1. The digit 9 is the closure ($\varphi(9)$ is read as 9).

2.2 The three chambers.

Chamber	Elements	Image under φ	Role among numbers $6k \pm 1$
Right rail	$\{1, 4, 7\}$	$\{8, 5, 2\}$, the left rail	candidates $6k + 1$
Left rail	$\{2, 5, 8\}$	$\{7, 4, 1\}$, the right rail	candidates $6k - 1$
Central axis	$\{3, 6, 9\}$	itself	multiples of 3

2.3 The digital root. $DR(n)$ is the repeated digit sum of n , a value in $\{1, \dots, 9\}$; equivalently, $DR(n)$ is 1 plus the remainder of $n - 1$ in the division by 9. It respects sums and products: $DR(a + b) = DR(DR(a) + DR(b))$ and $DR(ab) = DR(DR(a) \cdot DR(b))$. **Th.**

3. Theorem 1: compression onto the right rail

Theorem 1 (Th). For every positive integer n , $DR(3n + 1) \in \{1, 4, 7\}$. The compression is 3 to 1, and each fibre is one chamber.

d	1	2	3	4	5	6	7	8	9
$3d + 1$	4	7	10	13	16	19	22	25	28
DR	4	7	1	4	7	1	4	7	1

Proof. $DR(3n + 1) = DR(3 \cdot DR(n) + 1)$, and the table covers the nine cases. The fibre of 1 is $\{3, 6, 9\}$ (the axis, absorbed), the fibre of 4 is $\{1, 4, 7\}$ (the right rail folds onto itself), the fibre of 7 is $\{2, 5, 8\}$ (the left rail crosses). ■

Reading (L). After one strike, the memory of the original chamber is destroyed at the level of the address.

4. Theorem 2: rail alternation

Theorem 2 (Th). Halving multiplies the digital root by 5 (because $2 \times 5 = 10$ has root 1). Starting from the right rail, successive halvings alternate strictly between the rails with period 6: $1 \rightarrow 5 \rightarrow 7 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$. No element of $\{3, 6, 9\}$ appears.

4.1 (L) Compression and permutation. The strike sends nine positions onto three: irreversible. The halving sends six positions onto six: reversible. Each strike destroys the chamber identity; halvings cannot restore it. This reading holds at the level of addresses; it does not prove convergence (Section 8).

5. Theorem 3: absorption of the central axis

Theorem 3 (Th). Every orbit enters the rail system $\{1, 4, 7\} \cup \{2, 5, 8\}$ after at most one strike and stays there.

Proof. $DR(n) \in \{3, 6, 9\}$ gives $DR(3n + 1) = 1$ (Theorem 1). Halvings of the rail positions stay on the rails (Theorem 2), and a strike from a rail position lands on $\{4, 7\}$. ■ Checked on every orbit below 200 000.

6. Theorem 4: the attractor (1, 8)

6.1 Squaring on the pairs (Th).

Pair	$DR(x^2)$	$DR(y^2)$	Behaviour
(1, 8)	1	1	fixed point
(2, 7)	4	4	sent into (4, 5)
(4, 5)	7	7	sent into (2, 7)
(3, 6)	9	9	collapses to the closure

Theorem 4 (Th). The only Collatz cycle containing 1 is $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$. It runs through the three operational pairs by increasing span: 4 in (4, 5) of span 1, then 2 in (2, 7) of span 5, then 1 in (1, 8) of span 7. Only (1, 8) is fixed under squaring.

6.2 (L) Why (1, 8). It has the maximal span 7, with half-span 3.5 and $k_arc = 3.5/7 = \frac{1}{2}$; it is the only pair fixed under squaring; it holds the identity 1 and the penultimate $8 = 9 - 1$, straddling the whole alphabet. Section 12.6 shows that the 8 is also the obligatory gate *at the level of integers* (Theorem 26).

7. The convergence question, restated

7.1 Cycles (Th; open). Version 2.1 argued that since $\log_2 3$ is irrational, $3^k \neq 2^m$, and concluded that no non-trivial cycle exists. That argument is retracted: the irrationality only rules out the exact equality $3^k = 2^m$, while a cycle needs the exact balance of Theorem 25, $2^m = 3^k \cdot \Pi(1 + 1/(3n_i))$, where the product of the +1 corrections can close the gap. The exclusion of non-trivial cycles therefore stays open. What is known: no cycle of a repeated word up to 14 moves exists besides the trivial one (V4.9.11, Section 12.7); and (F) a non-trivial cycle would have to contain more than 7.2×10^{10} odd terms (Hercher, 2023).

7.2 Divergence (Th; open). After a strike, $v_2(3n + 1)$ halvings follow, and the net factor of the odd step is $3/2^v$. For $v = 1$ the step grows (factor $3/2$); for $v \geq 2$ it contracts (factor at most $3/4$). Which case occurs is fixed exactly by the quarter dichotomy (Theorem 21). Growth chains are unbounded (Theorem 22). Whether an orbit can meet growth steps often enough, forever, to escape is the open part: P-COL-1.

7.3 Information (retracted, replaced). Version 2.1 counted one bit lost per odd step and concluded that the root information is exhausted after about $\log n$ odd steps, quoting an empirical stopping time near $6.95 \cdot \log_2 n$. This is retracted: Theorem 23 proves that the digital root carries no bit on the valuation, so the loss of address information says nothing about the descent. What replaces it is exact: along an orbit

of L terms with k strikes and m halvings, $L = k + m + 1$ (Theorem 25), and $L < n$ for every n from 111 to 200 000 (Th, finite computation).

8. The bridge from address to magnitude

Each odd step is a strike followed by $v_2(3n + 1)$ halvings, with net factor $F = 3/2^v$. Version 2.1 wrote that “Section 12 proves” persistent $v = 1$ to be impossible. Section 12 of version 2.1 did not prove it, and it is false that the address can prove it: by Theorem 23, any combination of digital root and valuation occurs, at every depth. The bridge is precisely the work site:

P-COL-1 (P, with criterion). Universal convergence requires showing that the contraction of addresses (Theorems 1 to 3, the world of the brick 3) entails the contraction of magnitudes (the world of the brick 2) *for each individual orbit*, not only for the population of orbits taken globally. Status: open. *Criterion, falsifiable both ways:* any candidate proof issued from the ARC address system must cross the frontier of Theorem 23 explicitly and declare by which instrument, foreign to the addresses, it counts the bricks 2. A proof that does not declare it repeats the retracted error.

9. Theorem 5: halving and the powers of 2

Theorem 5 (Th). The period-6 cycle of the powers of 2 on the nine positions, 2, 4, 8, 7, 5, 1, is the halving cycle of Theorem 2, 5, 7, 8, 4, 2, 1, traversed in reverse: halving is the inverse of doubling. ■

Reading (L). Every constraint drawn from rail alternation is at the same time a constraint on the residues of the powers of 2 by 9. It is not a constraint on the valuation v_2 of a given integer (Theorem 23).

10. Theorem 6: the digital-root channel

Theorem 6 (Th for (a) to (d)). The digital root is (a) compatible with sums and products (it is the projection of the integers on the cycle of nine positions); (b) closed under the strike: $DR(3n + 1) = DR(3 DR(n) + 1)$; (c) closed under halving: $DR(n/2) = DR(5 DR(n))$; (d) non-trivial: it travels the rails.

Uniqueness (L, downgraded). Version 2.1 added that the digital root is *the unique* standard arithmetic channel with these properties: prime factorisation fails (b) ($3 \times 7 + 1 = 22 = 2 \times 11$, and no algebraic function sends the factors of 7 to those of 22), and binary writing fails (b) as well, the strike scrambling bits non-locally. This uniqueness is established only by comparison among three channels, without a general definition of “channel”. It is a reading.

11. Theorem 7: twin prime products

Theorem 7 (Th). For every twin pair $(p, p + 2)$ with $p > 3$, $DR(p(p + 2)) = 8$.

Proof. A twin pair beyond 3 is $(6k - 1, 6k + 1)$; the product is $36k^2 - 1$, and $36k^2$ is a multiple of 9. ■ Checked on the 1 223 twin pairs below 100 000.

Reading (L). Twin products sediment to root 8, the non-unit element of the attractor (1, 8). Collatz sees the pair from the side of 1; twins see it from the side of 8. Same object, the two sides of the mirror φ .

12. What replaces Theorem 8

12.1 What version 2.1 claimed, and why it fails. Theorem 8 of version 2.1 built a deterministic chain on the 18 odd classes of remainders by 36, with a table of fixed valuations per class (2, 4, 2, 3, 2, 6, 2, 3, 2 for the contraction classes, central value 2.889), a “maximum growth chain” of 3 steps ($15 \rightarrow 23 \rightarrow 35 \rightarrow 17$), a two-state chain ($6/9, 3/9, 5/9, 4/9$) with stationary weights $5/8$ and $3/8$, a contraction factor 0.662 per odd step (with the derived values $0.816, 3.375 \times 0.75 = 2.531$ and 1.261), and the renaming of the gap as “the mixing time of a finite chain”. Each quantity was computed on the *smallest representative* of its class and then engraved as a property of the whole class. This is a well-definedness error, and two integers refute it by name.

Sentinel 37 (Th). 37 and 1 leave the same remainder 1 in the division by 36. But $3 \times 1 + 1 = 4$ carries two factors 2, and $3 \times 37 + 1 = 112 = 2^4 \times 7$ carries four. The valuation is not a function of the class: it depends on binary digits that the reduction by 36 has thrown away. Nine of the eighteen odd classes are affected; the table of valuations is false.

Sentinel 31 (Th). $31 \rightarrow 47 \rightarrow 71 \rightarrow 107 \rightarrow 161$: four consecutive growth steps, where the table promised at most three. The family $2^k - 1$ grows $k - 1$ times: 63 grows 5 times, 255 seven times, $2^{20} - 1$ nineteen times. “Maximum chain 3” was true of the nine representatives tested and false of the orbits. This is why the orbit of 27 climbs to 9 232.

With them fall the table, the stationary weights $5/8$ and $3/8$ as exact values, the factor 0.662 and the renaming of the gap.

12.2 Theorem 21 (Th): the quarter dichotomy. For every odd n : $n = 4t + 3$ if and only if $v_2(3n + 1) = 1$ (growth step, factor $3/2$); $n = 4t + 1$ implies $v_2(3n + 1) \geq 2$ (contraction step, factor at most $3/4$). Exact, without exception; checked on every odd number below 200 000. This is the exact part of the former Theorem 8, stripped of its class apparatus: the 18 odd classes do split 9 and 9 between growth and contraction, but the *depth* of the contraction is not fixed by the class.

12.3 Theorem 22 (Th): growth chains, characterised. An orbit makes at least k consecutive growth steps from the odd number n if and only if $n + 1$ is a multiple of 2^{k+1} , that is, if the binary writing of n ends with $k + 1$ digits 1. Chains of every length occur (the family $2^k - 1$), so they are unbounded; their density is $2^{-(k+1)}$, each extra growth step halving the population able to make it. This is the true statement of which “maximum 3 steps” was the shadow on nine representatives. Checked for $k = 1$ to 10 on every odd number below 2^{16} .

12.4 Theorem 23 (Th): the strangeness of the two bricks. The address of an integer on the cycle of the brick 3 (its digital root, and with it the rails, the axis and the inversion) is independent of its charge in brick 2, the valuation $v_2(3n + 1)$ that governs the descent. Since $\gcd(9, 2^k) = 1$ for every k , every combination (root, valuation) occurs: the 54 cells (9 roots $\times$ valuations 1 to 6) are all filled, checked on the odd numbers below two million, and the filling extends to every depth by the Chinese remainder theorem. The digital root carries no bit of information on the depth of the divisions by 2.

Reading (L), Lecture 28. The brick 3 and the brick 2 are foreign to each other: the address, in the world of 3, cannot predict the descent, in the world of 2. Forcing one into the other has a price.

12.5 Theorem 24 (Th): the +1 returns the amputated half.

- (a) For every odd n , $(3n + 1)/2 = 3n/2 + 1/2$. Halving an odd number would amputate a half; the +1 placed before the cut is exactly that half, returned, and the result falls back on an integer. *"It returns; it does not descend: it is the half that descends."*
- (b) For every odd c , $3(cm) + c = c(3m + 1)$: the world of $3n + c$ contains an exact copy of the world of $3n + 1$, enlarged c times.
- (c) 1 is the only integer that divides every integer, so the world of +1 has only one copy of itself: one house. For $c > 1$, the numbers not divisible by c fall elsewhere: $3n + 7$ has two houses (cycles with smallest terms 5 and 7), checked on 1 to 20 000, while $3n + 1$ has one.

What the theorem says: why Collatz *can* come back down at each strike, and why there is a single possible arrival. What it does not say: that the descent *ends*.

12.6 Theorems 25 and 26 (Th): the exact balance and the gate 8.

Theorem 25. Along an orbit from n to 1 with k strikes (the odd terms $n_1, \dots, n_k$ met) and m halvings, exactly and in fractions,

$$2^m = 3^k \cdot n \cdot \prod (1 + 1/(3n_i)), \text{ and } L = k + m + 1,$$

where L is the number of terms. *Proof:* each strike multiplies n_i by $3(1 + 1/(3n_i))$, each halving divides by 2, and the product of all operations sends n to 1. ■ Checked exactly for $n = 1$ to 3 000. Example: $n = 3$ gives the orbit 3, 10, 5, 16, 8, 4, 2, 1, with $k = 2$, $m = 5$, $L = 8$, and $2^5 = 27 \times (10/9) \times (16/15) = 32$. The return to 1 is the moment when the halvings have caught up with the strikes. The balance links k , m and n once the path is travelled; it does not predict k from n .

Theorem 26. Every orbit starting outside $\{1, 2, 4, 8\}$ ends with $16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$, and 16 can only come from 32 (halving) or from 5 (strike). More generally, the last odd term of every path is of the form $(4^i - 1)/3$, that is 1, 5, 21, 85, 341, 1 365, ..., and the final descent is pure 2. Checked up to 200 000. (The source wrote "outside $\{1, 2, 4\}$ "; the orbit of 8 itself is the one exception, corrected here.) **Reading (L):** the 8 is the obligatory gate; the pair (1, 8), closed by 9, is crossed by every integer before it lands.

12.7 What the binary writing shows (Th).

V4.9.10, novelty always releases. An odd number whose binary writing ends with t digits 1 makes exactly $t - 1$ strikes with a single halving each, then at least two halvings come together. The extreme case is $2^p - 1$: it makes p compressed steps $(3n + 1)/2$, each replacing a factor 2 by a factor 3, and exits exactly on $3^p - 1$ (checked up to $p = 39$). Example: $31 \rightarrow 47 \rightarrow 71 \rightarrow 107 \rightarrow 161 \rightarrow 242 = 2 \times 11^2$. Every positive integer ends with a finite run of 1s, so every run of growth is released in finite time.

V4.9.11, the only rising word that repeats lives below zero. A word of moves repeated without end has a single seed, $c / (2^L - 3^k)$. If the word rises ($3^k > 2^L$), the seed is below zero. Exact computation: in the alphabet {strike-and-halve, halve}, the 1 767 rising words of up to 12 moves all have their seed below zero, and among the falling words the only positive integer seeds are 1 and 2, the trivial cycle (the corpus ran the same computation up to 14 moves). The word "always rise" has seed -1 , the number

whose binary digits are all 1s, the only fixed point of $x \mapsto (3x + 1)/2$. The Mersenne number $2^p - 1$ is that -1 cut to p digits: it rises as long as it carries the piece, then exits.

V4.9.12 (Th; F), the finite window is not enough. Every finite sequence of strikes and halvings is realised by exactly one number in each block of 2^L integers (Terras, 1976; redone here for $L = 12$, 4 096 words on 4 096 numbers). No rule bearing on a finite number of steps can therefore forbid a long climb: the knot is global.

12.8 Corollary (Th): no finite archive closes. For every bound K there are infinitely many odd n whose orbit does not pass below n during its first K accelerated steps. Hence an archive of finitely many residue-class certificates, each attesting a descent within at most K steps, leaves infinitely many integers without a certificate.

Proof. Take n with $n + 1$ a multiple of $2^{(K+1)}$, for instance $n = 7 \cdot 2^{(K+1)} - 1$. By Theorem 22 the first K steps are all growth steps $y \mapsto (3y + 1)/2$, and after k of them, exactly, $y(k) = (3/2)^k (n + 1) - 1$, which is strictly greater than n for every k from 1 to K . A finite archive has a longest certificate; take K equal to its length. ■ Checked in exact integers for $K = 10, 50, 250$ and $1\,000$.

What the corollary does not exclude: a finite archive *completed by an analytic argument* that treats arbitrarily long climbs separately. It says that the finite part can never suffice alone. Version 2.1's Theorem 8 was such a finite part; so is any table of classes by a fixed modulus of 2.

12.9 (L) The lesson. The cause of the failure of Theorem 8 is of the same family as the gesture the corpus reproaches in the identification of 9 with 0: “*reading on a finite projection a property that lived in what the projection had amputated.*” The $+1$ returns at each strike the half the cut would have thrown (Theorem 24), and the run of 1s that makes an orbit climb can always be longer than what a fixed depth has foreseen (Theorem 22). A finite depth of powers of 2 cannot contain all the climbs: the 3 does not fit into the 2. “*What must be sought is not the 1, it is the 2: how many times it passes.*”

12.10 (Th; F) The forced case. Replacing 3 by 5 breaks the problem: $5n + 1$ has parasitic cycles, $13 \rightarrow 66 \rightarrow 33 \rightarrow 166 \rightarrow 83 \rightarrow 416 \rightarrow 208 \rightarrow 104 \rightarrow 52 \rightarrow 26 \rightarrow 13$ and $17 \rightarrow 86 \rightarrow 43 \rightarrow 216 \rightarrow 108 \rightarrow 54 \rightarrow 27 \rightarrow 136 \rightarrow 68 \rightarrow 34 \rightarrow 17$. *Criterion:* a proof that would also hold for $5n + 1$ is false; it must use what is proper to the 3, as P-COL-1 requires.

13. Worked examples

13.1 The orbit of 27 (Th). 27 has root 9 (central axis). The first strike gives 82, root 1: the axis is absorbed (Theorem 3). $27 = 4 \times 6 + 3$, so the step grows (Theorem 21): $82/2 = 41$. Then $41 = 4 \times 10 + 1$, so the next step contracts: $3 \times 41 + 1 = 124 = 2^2 \times 31$, and $v_2 = 2$. Then $31 = 2^5 - 1$ starts a growth chain of four steps (Theorem 22, sentinel 31). The orbit takes 111 steps, climbs to 9 232, and makes 41 strikes and 70 halvings, so $L = 112 = 41 + 70 + 1$ (Theorem 25).

13.2 Conway's variation (L). Conway's “amusical” variation is invertible and admits non-trivial cycles. The Collatz map is not invertible: many numbers reach the same value. “*Where information is preserved, cycles can form. Where information is destroyed, only descent is possible.*” This is a reading at the level of addresses; it does not prove convergence (Section 8).

14. Status and falsifiability

14.1 Proved.

Statement	Status
Theorem 1: compression 3:1 onto the right rail	Th
Theorem 2: rail alternation, period 6	Th
Theorem 3: absorption of the central axis	Th
Theorem 4: terminal cycle $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$ at (1, 8)	Th
Theorem 5: halving is doubling reversed on the nine positions	Th
Theorem 6 (a) to (d); uniqueness of the channel	Th; L
Theorem 7: twin products have root 8	Th
Theorem 8 of version 2.1	Retracted
Theorems 21 to 26, V4.9.10 to V4.9.12, Corollary 12.8	Th
Lecture 28, readings 4.1, 6.2, 12.9, 13.2	L
Convergence of every orbit	P-COL-1, open

14.2 What remains (Chantier). Proved: the address system (Theorems 1 to 5), the release in finite time of every run of 1s (V4.9.10), the absence of a repeating rising word for a positive integer (V4.9.11), the exact balance (Theorem 25). One case remains: an infinite word that rises without ever repeating. One would have to show that a positive integer cannot recreate without end runs of 1s long enough to push back each release. This is the knot of the conjecture, named in the language of the addresses; P-COL-1 keeps its criterion.

14.3 Falsification criteria, after the retraction.

- F1: an integer whose orbit does not reach 1 (would refute the conjecture; valid).
- F2: a non-trivial Collatz cycle (valid).
- F3: a compression-permutation system of the same structure that admits a non-trivial cycle (valid for reading 4.1).
- F4: a twin pair beyond 3 whose product does not have root 8 (impossible: Theorem 7).
- F5: a demonstration that the transition graph on the classes by 36 does not constrain the statistics of v_2 (reached by the retraction: criterion met, statement withdrawn).

15. Conclusion

Seven theorems of version 2.1 stand. The eighth is withdrawn by name, and in its place stand the quarter dichotomy, the unbounded chains, the strangeness of the bricks 3 and 2, the returned half, the exact balance, the gate 8, and the proof that no finite archive of classes can close the question alone. The conjecture is open, and the place where it is open is now named: the passage from the address to the count of the 2.

The closing lines of version 2.1 are kept as the author's sentences. Their mathematical content is P-COL-1, open.

Numbers fall to 1 for the same reason light sediments to iron: there is no other resting place.

Every number returns to 1. Not by accident. Not by probability. By the irreversible geometry of nine digits.

References

1. Collatz, L. (1937). Unpublished problem, University of Hamburg.
2. Terras, R. (1976). A stopping time problem on the positive integers. *Acta Arithmetica* 30, 241-252.
3. Lagarias, J. C. (1985). The $3x + 1$ problem and its generalizations. *American Mathematical Monthly* 92, 3-23.
4. Lagarias, J. C., ed. (2010). *The Ultimate Challenge: The $3x + 1$ Problem*. AMS.
5. Tao, T. (2022). Almost all orbits of the Collatz map attain almost bounded values. *Forum of Mathematics, Pi* 10, e12 (preprint arXiv:1909.03562, 2019).
6. Barina, D. (2021). Convergence verification of the Collatz problem. *The Journal of Supercomputing* 77, 2681-2688; verification since extended to 695×2^{60} .
7. Hercher, C. (2023). There are no Collatz m-cycles with $m \leq 91$. *Journal of Integer Sequences* 26.
8. Simons, J. and de Weger, B. (2005). Theoretical and computational bounds for m-cycles of the $3n + 1$ problem. *Acta Arithmetica* 117, 51-70.
9. Conway, J. H. (1972). Unpredictable iterations. *Proceedings of the 1972 Number Theory Conference*, Boulder, 49-52.
10. Erdős, P., quoted in Lagarias (1985).
11. Bastos, F. (2026). *La Théorie des nombres*, Volume 4 of the ARC corpus, Part III (chapters 8 to 10) and Appendix B (retractions).
12. Bastos, F. (2026). *Collatz : pourquoi aucune archive finie ne ferme*, ARC corpus, version 1.0.

Contact: contact@d2technologie.com

Modèle, questions, lectures et règles : Filipe Bastos. Scribe : Claude, sous la contradiction de l'auteur. Calculs, formulations et audits assistés par IA (ChatGPT, Gemini, DeepSeek, GLM), à sa demande.

Corpus ARC, Filipe Bastos, CC BY 4.0.