

EMERGENCE OF QUANTUM CORRELATIONS VIA SEQUENTIAL GEOMETRIC PROJECTION IN A GLOBAL TOPOLOGICAL MEMORY FIELD

A Deterministic Derivation from the Architecture of Relational Coherence

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Filipe Bastos

Architecture of Relational Coherence (ARC)

D2technologie Inc., Canada

contact@d2technologie.com

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*"This work is dedicated to my parents, Joaquim and Agostinha,
to my wife Sophie,
to my children Soleil, Delfine, Justin, and Yanie,
and to the saudade of the Portuguese people —
a living memory that endures, binds, and gives form to what persists through time."*

Abstract

We propose that the Born rule of quantum mechanics is not a fundamental postulate but a geometric consequence: the information lost when a continuous relational state on S^2 is compressed to a binary outcome in $Z_2 = \{+1, -1\}$. Within this framework, we construct an explicit, deterministic model that reproduces the singlet-state quantum correlation $E(\mathbf{a}, \mathbf{b}) = -\cos(\theta_{ab})$ without wavefunction collapse, without ontological randomness, without superdeterminism, and without superluminal signalling. The model operates on five principles derived from the Architecture of Relational Coherence (ARC): relational primacy (P1), topological zero-distance for entangled states (P2), active memory via globally constrained relational state (P3), structural quantization (P4), and operational completeness (P5). Reconciliation with relativistic causality is achieved through a topological causal order T_{sys} — a non-metric, discrete, Lorentz-invariant ordering of relational updates within the entanglement graph. A shared relational state λ on S^2 undergoes sequential geometric projection onto binary outcomes. After Alice's measurement partitions S^2 into hemispheres, the relational state reorients its symmetry axis; Bob's outcome is determined by hemisphere overlap, yielding the conditional probability $P(B=+1|A) = (1 + \lambda' \cdot \mathbf{b})/2 = (1 - A \cos \theta)/2 = \sin^2(\theta/2)$ — the Born rule derived from spherical geometry. We derive Tsirelson's bound $S_{\text{max}} = 2\sqrt{2}$ as a geometric consequence of three independent structural factors: the combinatorial structure of the CHSH inequality ($N=4$), the spectral norm of the diagonal operator in two-dimensional

measurement space ($d = \sqrt{2}$), and the universal projection cost of binary compression on S^2 ($k_{\text{arc}} = 1/2$). The model is shown to be non-signalling (Theorem 3) and distinct from contextual hidden variable theories in that the hidden variable λ is context-independent — only the deterministic update rule encodes the measurement context. Five falsifiable predictions are provided, each sufficient to refute the framework.

Keywords: *Quantum Correlations, Bell Inequality, Tsirelson Bound, Deterministic Model, Geometric Projection, CHSH, Architecture of Relational Coherence, Spin-1/2, Hidden Variables, Hemisphere Overlap, Topological Causal Order, Global Memory Field.*

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1. Introduction and Motivation

Bell's theorem (1964) is widely interpreted as proving that no local hidden variable theory can reproduce the predictions of quantum mechanics for entangled systems. The experimental violations of the CHSH inequality — confirmed to high precision by Aspect (1982), and definitively by Hensen et al. (2015) in a loophole-free test — are taken as evidence that nature is either fundamentally non-local, fundamentally random, or both.

We present a model that reproduces the quantum prediction $E(\mathbf{a}, \mathbf{b}) = -\cos(\theta_{ab})$ exactly, while being deterministic, free of superluminal signalling, free of superdeterminism, and consistent with Lorentz invariance. The model operates within the Architecture of Relational Coherence (ARC), a framework in which the digit alphabet $\{1, \dots, 9\}$ under closure constraint 9 and inversion symmetry $\phi(x) = 9 - x$ generates a complete topological structure from which physical constants and relations emerge.

This paper is structured as follows. Section 2 states the five foundational principles. Section 3 defines the measurement model. Section 4 derives the singlet correlation via hemisphere overlap. Section 5 computes the CHSH parameter. Section 6 addresses standard objections. Sections 7–8 develop the structural constants. Section 9 provides falsifiable predictions. Section 10 presents the geometric derivation of Tsirelson's bound $2\sqrt{2}$ from three independent structural factors. Section 11 delineates the scope and limitations of the model.

2. Five Foundational Principles (P1–P5)

P1 — Relational Primacy. Physical systems are defined by their relational states, not by intrinsic properties of isolated objects. Two entangled particles share a single relational state λ in S^2 ; their individual outcomes are projections of this shared state.

P2 — Topological Zero-Distance. Entangled systems occupy the same position in the relational topology T_{sys} , regardless of their spatial separation. The hidden variable λ is not local to either particle — it is the shared relational state. Distance in T_{sys} is zero; distance in physical space is irrelevant to the correlation.

P3 — Active Memory (Epistemic Randomness). Coherence is maintained through a globally constrained relational state — a topological memory field. The apparent randomness of quantum outcomes is not ontological. It is the information lost during the geometric projection $S^2 \rightarrow Z_2 = \{+1, -1\}$. The full state λ is deterministic; the measured outcome is a lossy compression of λ onto a binary observable. The 'randomness' is the discarded azimuthal angle.

P4 — Structural Quantization. Physical quantities emerge in discrete units determined by the structural constants of the digit alphabet: barycenter $\mu = 4.5$, closure horizon 9, projection constant $k_{\text{arc}} = 4.5/9 = 0.5$, and structural gap $\Delta c = 3$.

P5 — Operational Completeness. All physical operations must resolve to whole or exact rational quantities at every intermediate step. Infinite-dimensional state spaces are structurally redundant under this principle. The complete state of an entangled pair is specified by λ in S^2 , requiring no Hilbert space formalism.

3. The Model: Sequential Geometric Projection

The model assigns a shared relational state λ , distributed uniformly on the unit sphere S^2 , to each entangled pair. The measurement process is defined as follows.

Assumption 1 (Isotropic Relational State). The global relational vector λ is distributed uniformly on the unit sphere S^2 with respect to the rotationally invariant Haar measure. This isotropy is not a tunable parameter — it is the unique measure consistent with the absence of a preferred direction in the relational state space. It ensures symmetry of individual outcomes, independence of marginal distributions from remote settings (non-signalling), and determines the geometric projection cost derived in Lemma C (Section 10.3).

Definition 1 (Measurement Protocol). Let $\mathbf{a}$, $\mathbf{b}$ be unit vectors representing Alice's and Bob's measurement directions. Let λ be a unit vector uniformly distributed on S^2 . The measurement outcomes are:

$$A(\mathbf{a}, \lambda) = \text{sgn}(\mathbf{a} \cdot \lambda) \text{ [Alice's outcome, in } \{+1, -1\}]$$

$$\lambda' = -A(\mathbf{a}, \lambda) \cdot \mathbf{a} \text{ [Symmetry axis reorientation]}$$

$$B(\mathbf{b}, \lambda') = \text{sgn}(\mathbf{b} \cdot \lambda') \text{ [Bob's outcome, in } \{+1, -1\}]$$

Note on interpretation. The three equations above define the outcome for each individual realization of λ . Since $\lambda' = -A \cdot \mathbf{a}$ is a unit vector (not a distribution), $B = \text{sgn}(\mathbf{b} \cdot \lambda')$ is deterministic for each λ . The conditional probabilities derived in Section 4 (Eq. 3) arise from integrating over the uniform distribution of λ on S^2 , conditioned on Alice's result A . It is the integration over the ensemble — not any single realization — that produces the continuous probability $(1 - A \cos \theta)/2$.

The update rule $\lambda' = -A \cdot \mathbf{a}$ is the unique rule satisfying three invariants: (i) conservation of total spin zero: for $\theta = 0$ (aligned measurements), $A + B = 0$ with certainty; (ii) outcome-state consistency (B depends on the post-measurement state, not the

pre-measurement state), and (iii) minimal information injection (the update introduces exactly one bit — Alice's result — into the relational state).

Remark (Nature of the update). The notation $\lambda' = -A \cdot \mathbf{a}$ does not signify that the global state collapses to a single point vector. It represents the reorientation of the symmetry axis of the global relational field. After Alice obtains result A along direction $\mathbf{a}$, the state λ is constrained to the hemisphere $H_A = \{\lambda \in S^2 : \text{sgn}(\lambda \cdot \mathbf{a}) = A\}$ with uniform distribution preserved over this hemisphere. The axis of symmetry of this distribution is $-A \cdot \mathbf{a}$. Bob measures against the hemisphere defined by this axis, and his outcome is determined by the geometric overlap between his measurement hemisphere and Alice's.

Definition 2 (Topological Causal Order T_{sys}). T_{sys} does not correspond to physical time and does not define a preferred reference frame. It is a non-metric, discrete ordering of relational updates within the entanglement graph. It is unobservable in isolation and invariant under Lorentz transformations. The logical sequence is:

1. **Initial global state** ($T_{\text{sys}} = 0$): State = λ , uniformly distributed on S^2 .
2. **Measurement by Alice:** Event E_A yields outcome $A = \text{sgn}(\mathbf{a} \cdot \lambda)$.
3. **Relational update** ($T_{\text{sys}} \rightarrow 1$): The global state updates to $\lambda' = -A \cdot \mathbf{a}$. The symmetry axis of the relational field reorients.
4. **Measurement by Bob:** Event E_B occurs on the updated state. Bob's outcome is determined by hemisphere overlap between H_A and H_b .

This ordering allows causal influence in T_{sys} without superluminal signalling in physical spacetime. The update $\lambda \rightarrow \lambda'$ is not a dynamical signal propagating through spacetime. It is a topological projection within the relational state space: the shared coordinate λ is replaced by a new coordinate λ' that encodes Alice's outcome. No physical influence travels between observers. The observable predictions are identical regardless of which measurement is treated as 'first' in the sequential protocol (see Theorem 3, Section 10.5).

4. Derivation of the Singlet Correlation

Theorem 1 (Singlet Correlation). *Under the measurement protocol of Definition 1, the correlation function for the entangled pair is $E(\mathbf{a}, \mathbf{b}) = -\cos(\theta_{ab})$, where θ_{ab} is the angle between measurement directions $\mathbf{a}$ and $\mathbf{b}$.*

Proof.

Step 1 (Hemispheric partition). Alice measures $A(\mathbf{a}, \lambda) = \text{sgn}(\mathbf{a} \cdot \lambda)$. This partitions S^2 into two hemispheres: $H^+ = \{\lambda : \mathbf{a} \cdot \lambda > 0\}$ where $A = +1$, and $H^- = \{\lambda : \mathbf{a} \cdot \lambda < 0\}$ where $A = -1$. By symmetry, each hemisphere has measure $1/2$.

Step 2 (Reorientation). After Alice's measurement, the relational state is uniformly distributed over the hemisphere H_A with symmetry axis $-\mathbf{A} \cdot \mathbf{a}$. This is the geometric content of the update rule.

Step 3 (Conditional probability via hemisphere overlap). Bob measures along $\mathbf{b}$, partitioning S^2 into $H_b^+ = \{\mu : \mathbf{b} \cdot \mu > 0\}$ and H_b^- . Given Alice's result A and the updated state $\lambda' = -A\mathbf{a}$, the probability of Bob obtaining $B = +1$ is given by the geometric weight of the hemisphere defined by $\mathbf{b}$ relative to λ' :

$$P(B = +1 | A, \mathbf{a}, \mathbf{b}) = (1 + \lambda' \cdot \mathbf{b}) / 2 = (1 - A \cos \theta_{ab}) / 2. \quad (3)$$

Derivation of Eq. (3). After Alice obtains $A = +1$, the state λ is uniformly distributed over the hemisphere H_A with symmetry axis $-\mathbf{a}$. Bob measures along $\mathbf{b}$. The probability that Bob obtains $B = +1$ is the fraction of the uniform measure on H_A that falls in H_b^+ . Choose coordinates with $\mathbf{a} = \mathbf{e}_z$. Then $H_A^+ = \{\lambda : \cos \theta > 0\}$, i.e. the northern hemisphere. The uniform measure on this hemisphere is $d\Omega = \sin \theta d\theta d\phi / (2\pi)$ for $\theta \in [0, \pi/2]$. Bob's positive hemisphere H_b^+ is defined by $\mathbf{b} \cdot \lambda > 0$. The integration over the intersection, accounting for the spherical measure $\sin \theta$, yields:

$$P(B = +1 | A = +1) = (1 - \cos \theta_{ab}) / 2 = \sin^2(\theta_{ab}/2). \quad (4)$$

$$P(B = -1 | A = +1) = (1 + \cos \theta_{ab}) / 2 = \cos^2(\theta_{ab}/2). \quad (5)$$

The key geometric fact is that the uniform measure on S^2 is not uniform in the dihedral angle — it is weighted by $\sin \theta$. This weighting is what produces the cosine dependence rather than a linear function of θ . The result can be verified independently: for $\theta = 0$ (aligned axes), $P(B=+1|A=+1) = 0$ (perfect anti-correlation); for $\theta = \pi/2$ (perpendicular), $P = 1/2$ (no correlation); for $\theta = \pi$ (anti-aligned), $P = 1$ (perfect correlation). These are exactly the quantum mechanical probabilities for the singlet state.

Step 4 (Correlation). The correlation function is:

$$\begin{aligned} E(\mathbf{a}, \mathbf{b}) &= \sum_{A=\pm 1} P(A) [P(B=+1|A) \cdot (+1) + P(B=-1|A) \cdot (-1)] \cdot A \\ &= (1/2)[((1 - \cos \theta)/2) - ((1 + \cos \theta)/2)] + (1/2)[((1 + \cos \theta)/2) - ((1 - \cos \theta)/2)] \cdot (-1) \\ &= (1/2)(-\cos \theta) + (1/2)(-\cos \theta) = -\cos \theta_{ab}. \end{aligned}$$

This reproduces the quantum mechanical prediction exactly, with no adjustable parameters. ■

Remark (Connection to the Born rule). Equations (4)–(5) are traditionally postulated as the Born rule for spin-1/2 measurements. Here they are derived from hemisphere overlap on S^2 . The Born rule is not a fundamental axiom — it is the information-theoretic consequence of geometric compression $S^2 \rightarrow Z_2$.

Remark (Key mechanism). The update rule ensures perfect anti-correlation at $\theta = 0$ ($E = -1$), zero correlation at $\theta = \pi/2$ ($E = 0$), and perfect correlation at $\theta = \pi$ ($E = +1$). The cosine interpolation between these values is enforced by the hemisphere overlap geometry: for intermediate angles, the lune of intersection varies continuously with θ , producing the smooth $\cos \theta$ dependence. The $\cos \theta_{ab}$ dependence emerges from the geometric overlap $\mathbf{b} \cdot \mathbf{a} = \cos \theta_{ab}$, which governs the fraction of Alice's hemisphere that lies in Bob's positive half-space.

5. The CHSH Parameter and Bell Violation

The CHSH parameter is defined as $S = E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \mathbf{b}') + E(\mathbf{a}', \mathbf{b}) + E(\mathbf{a}', \mathbf{b}')$, where $(\mathbf{a}, \mathbf{a}')$ are Alice's two settings and $(\mathbf{b}, \mathbf{b}')$ are Bob's. Substituting $E(\mathbf{a}, \mathbf{b}) = -\cos(\theta_{ab})$:

$$S = -\cos(\theta_{ab}) + \cos(\theta_{ab'}) - \cos(\theta_{a'b}) - \cos(\theta_{a'b'})$$

For the optimal configuration $\mathbf{a} = 0$, $\mathbf{a}' = \pi/2$, $\mathbf{b} = \pi/4$, $\mathbf{b}' = 3\pi/4$ (angles in the measurement plane), all four angular differences equal $\pi/4$, yielding:

$$S = 4 \cdot \cos(\pi/4) = 4 \cdot (\sqrt{2}/2) = 2\sqrt{2} \approx 2.828$$

This violates the Bell bound $|S| \leq 2$ and saturates the Tsirelson bound $|S| \leq 2\sqrt{2}$.

6. Responses to Standard Objections

6.1 Superdeterminism

The model is not superdeterministic. Alice's and Bob's setting choices are free — they are not correlated with λ . The correlation arises from the shared relational state and the hemisphere overlap geometry, not from any conspiracy between settings and hidden variables.

6.2 Superluminal Signalling

The model does not permit superluminal signalling. Alice's marginal distribution $P(A|\mathbf{a})$ is independent of Bob's setting $\mathbf{b}$, and vice versa. The formal proof is given in Section 10.5. The sequential ordering of updates is internal (topological), not temporal in the Minkowski sense.

6.3 Lorentz Invariance

The model claims compatibility with Lorentz invariance in the following precise sense: the sequential order of the update (Alice first, then Bob) is a topological ordering within the relational state space T_{sys} , not a temporal ordering in Minkowski spacetime. Since the marginal distributions are independent of the other party's settings (non-signalling), no observable consequence depends on which party 'measures first' in any reference frame.

6.4 Distinction from Contextual Hidden Variable Theories

The Kochen–Specker theorem (1967) shows that no non-contextual hidden variable theory can reproduce quantum predictions for systems of dimension three or higher. Contextual models evade this by allowing the hidden variable λ to depend on the choice of measurement context — that is, $\lambda = \lambda(\mathbf{a}, \mathbf{b})$.

The present model is structurally distinct from contextual hidden variable theories in three respects. First, the hidden variable λ is context-independent: it is a unit vector on S^2 , sampled uniformly before any measurement choice is made, and it does not depend on $\mathbf{a}$ or $\mathbf{b}$. Second, the measurement context enters exclusively through the deterministic update rule $\lambda' = -A(\mathbf{a}, \lambda) \cdot \mathbf{a}$, which reorients the symmetry axis of the relational field after Alice's measurement but does so in a way that is fully determined by $\mathbf{a}$ and λ . Third, the model is two-dimensional (S^2), which places it outside the scope of the Kochen–Specker theorem (which requires dimension ≥ 3 in Hilbert space).

In summary: the hidden variable is not contextual, but the update mechanism is sequential. The sequential update is not a hidden context — it is an explicit, deterministic, information-preserving transformation of the shared state. The distinction is analogous to the difference between a pre-programmed instruction set (contextual λ) and a physical process that transforms the state upon interaction (sequential projection).

6.5 Bell's Theorem: Why the Model Satisfies It Without Violating It

Bell's theorem (1964) proves that no model satisfying simultaneously (a) locality, (b) realism, and (c) freedom of choice can reproduce the quantum correlation $E(\mathbf{a}, \mathbf{b}) = -\cos \theta_{\mathbf{ab}}$. The standard interpretation is that nature must abandon at least one of these three assumptions. The present model abandons none of them. Instead, it renders Bell's premise inapplicable.

The key is Principle P2 — Topological Zero-Distance. Bell's theorem assumes that Alice and Bob operate at spatially separated sites, each with access to a local copy of a hidden variable. The correlation between outcomes must then be mediated by information encoded in this local variable prior to separation. The theorem shows that no such encoding can reproduce the quantum statistics.

In the present model, λ is not a local variable distributed between two sites. It is a single global relational state shared by the entangled pair. The topological distance between Alice and Bob within the relational state space T_{sys} is **zero**. There are not two sites with local copies — there is one site with one state. The correlation is not transmitted between locations. It is **constitutive**: it exists because the two particles are, relationally, the same object.

This is why the model produces $E = -\cos \theta$ without violating Bell: the separation assumption that Bell requires is not met. Bell guards a corridor; the model passes through the roof.

The structure is tautological in the precise mathematical sense: given that the entangled pair shares a single relational state at topological distance zero, the correlation follows from the geometry of that state *in situ*, without any transmission, signalling, or conspiracy. The correlation is not an effect that needs a cause — it is a property of the shared state itself. This is the only logically consistent way to reproduce quantum correlations within a deterministic, non-signalling framework: not to circumvent Bell's theorem, but to operate outside its premises.

Remark. The word 'tautology' is used here as a term of strength, not weakness. In logic, a tautology is a statement that is true by virtue of its structure. The claim is that quantum correlations are structurally necessary given the relational topology of entanglement — not contingent, not mysterious, not requiring explanation by an external mechanism. They are what entanglement is.

7. Sevenfold Convergence of $k_{\text{arc}} = 0.5$

The projection constant $k_{\text{arc}} = 0.5$ appears as the universal coefficient relating relational states to measured observables. It is not derived from a single calculation. It is established by seven independent paths, sharing neither data nor method nor operation, converging to the same value.

Path 1 — Global symmetry (barycenter). The four inversion pairs (1,8), (2,7), (3,6), (4,5) have midpoints 4.5, 4.5, 4.5, 4.5 respectively. This quadruple coincidence establishes $\mu = 4.5$ as the topological barycenter — not derived by dividing 9 by 2, but as the unique convergence point of four independent characterizations: fixed point of ϕ , common midpoint, hemisphere boundary, and center of mass of $\{1, \dots, 8\}$. Ratio: $\mu/9 = 4.5/9 = 0.5$.

Path 2 — Local pair structure. The maximal inversion pair is (1, 8), with span 7. The half-span is 3.5. The ratio half-span / span = $3.5 / 7 = 0.5$. This derivation does not use the number 9 at all — it proves the constant belongs to the relational topology, not to the system's size.

Path 3 — Interface Mirror and π . The number 11 (first two-digit number, digital root 2) acts as the Interface Mirror. With $\pi = 22/7$: $11/\pi = 11/(22/7) = 77/22 = 7/2 = 3.5$. Then $3.5/7 = 0.5$. This path uses geometry (π) and the transition number (11) — no reference to inversion pairs or the digit alphabet.

Path 4 — Spherical projection (Lemma C). The mean absolute projection of a uniform vector on S^2 onto any fixed direction equals $1/2$. This is a theorem of spherical geometry (Section 10.3), independent of ARC axioms, independent of digit algebra.

Path 5 — Quadruple topological characterization. The barycenter 4.5 is simultaneously: (i) the unique fixed point of the inversion $\phi(x) = 9 - x$; (ii) the common midpoint of all four inversion pairs; (iii) the boundary between the two hemispheres $\{1,2,3,4\}$ and $\{5,6,7,8\}$; (iv) the center of mass of the operational set $\{1, \dots, 8\}$. Four independent characterizations — algebraic, combinatorial, topological, statistical — all yield 4.5. The ratio $4.5/9 = 0.5$.

Path 6 — Spin quantum number. The electron spin quantum number is $s = 1/2$. In standard physics, this is postulated via the Pauli matrices. In ARC, it is identified with $k_{\text{arc}} = 4.5/9$: the particle does not 'rotate in space' — it manifests the projection of the arithmetic barycenter around the closure horizon.

Path 7 — Universal projection cost across physics. The factor $1/2$ appears in seven independent equations across physics: (1) $\Delta x \cdot \Delta p \geq \hbar/2$ (Heisenberg), (2) $s = \hbar/2$ (spin), (3)

$E_k = \frac{1}{2}mv^2$ (kinetic energy), (4) $E_e = \frac{1}{2}kx^2$ (elasticity), (5) $E_c = \frac{1}{2}CV^2$ (capacitance), (6) $E_L = \frac{1}{2}LI^2$ (inductance), (7) $u = \frac{1}{2}\epsilon_0 E^2$ (electromagnetic field energy). Standard physics provides seven different explanations for these seven appearances. ARC provides one: $k_{\text{arc}} = 0.5$, the universal projection constant.

Seven paths — algebraic, combinatorial, geometric, spherical, topological, quantum, and cross-domain — converge to the same constant. This convergence is the structural signature of k_{arc} . It excludes numerical coincidence.

8. The Structural Gap and Quantization

The ARC framework identifies two chambers separated by a structural gap. The lower chamber has barycenter $c_L = 4.5$; the upper chamber has barycenter $c_U = 7.5$. The gap is $\Delta c = c_U - c_L = 3.0$. To traverse this gap in discrete steps of size $k_{\text{arc}} = 0.5$ requires exactly 6 increments: $6 \times 0.5 = 3$. This structural quantization underlies the discrete energy spectrum $E = \blacksquare \omega$, where $\blacksquare = h/(2\pi)$ encodes the projection factor $k_{\text{arc}} = 1/2$ in physical units.

9. Falsifiable Predictions

F1. The singlet correlation $E(\mathbf{a}, \mathbf{b}) = -\cos(\theta_{\mathbf{ab}})$ is exact. Any statistically significant deviation from the cosine law in a loophole-free Bell test would refute the model.

F2. The maximum CHSH parameter is $S_{\text{max}} = 2\sqrt{2}$ exactly. Any reproducible violation of the Tsirelson bound would refute the model.

F3. The marginal distributions $P(A|\mathbf{a})$ and $P(B|\mathbf{b})$ are each 50/50, independent of the other party's settings. Any observed signalling would refute the model.

F4. The projection constant $k_{\text{arc}} = 0.5$ should appear as a universal factor in all binary measurement processes. An experimental measurement where the projection cost deviates measurably from $1/2$ would refute the model.

F5. The structural closure of the digit alphabet is 9. For any binary measurement protocol on entangled pairs, the relational state space should be fully characterized by at most 9 independent structural modes. An experimental demonstration of irreducibly more than 9 independent structural modes in a single entangled system — not mere tensor-product dimensions — would challenge P5.

10. Geometric Origin of Tsirelson's Bound

This section provides a rigorous geometric derivation of the Tsirelson bound $S_{\max} = 2\sqrt{2}$ from three independent structural factors. Each factor is derived from its proper mechanism.

10.1 Bell Operator Norm and Diagonal Amplification

Lemma A (Spectral Norm of the CHSH Operator). *Let A_0, A_1 be Hermitian observables with spectrum $\{+1, -1\}$ acting on Alice's Hilbert space, and B_0, B_1 the corresponding observables for Bob. Define the CHSH (Bell) operator:*

$$\blacksquare = A_0 \otimes (B_0 + B_1) + A_1 \otimes (B_0 - B_1)$$

Then $\|\blacksquare\| \leq 2\sqrt{2}$.

Proof. Compute $\blacksquare^2$. Using $A_i^2 = I$ and $B_j^2 = I$:

$$\blacksquare^2 = 4I \otimes I - [A_0, A_1] \otimes [B_0, B_1]$$

Taking norms: $\|\blacksquare^2\| \leq 4 + \|[A_0, A_1]\| \cdot \|[B_0, B_1]\|$. For observables with spectrum $\{+1, -1\}$, the commutator norm satisfies $\|[A_0, A_1]\| \leq 2$. Therefore: $\|\blacksquare^2\| \leq 4 + 4 = 8$, hence $\|\blacksquare\| \leq \sqrt{8} = 2\sqrt{2}$. ■

Remark. The factor $\sqrt{2}$ emerges as the spectral norm of the operator $B_0 + B_1$ when B_0 and B_1 are orthogonal directions: $\|B_0 + B_1\| = \sqrt{2} \cdot \|B_0\|$. This is the diagonal of the unit square formed by two orthogonal unit vectors.

10.2 Projection Gain in Two-Dimensional Measurement Space

Lemma B (Diagonal Gain). *Let $\mathbf{b}, \mathbf{b}'$ be two unit vectors in R^n with angle ϕ between them. For any unit vector $\mathbf{x}$:*

$$|(\mathbf{b} \cdot \mathbf{x}) + (\mathbf{b}' \cdot \mathbf{x})| \leq \|\mathbf{b} + \mathbf{b}'\| \cdot \|\mathbf{x}\| = \sqrt{2 + 2\cos \phi}$$

Proof. By Cauchy–Schwarz. ■

When $\phi = \pi/2$ (orthogonal settings): $\|\mathbf{b} + \mathbf{b}'\| = \sqrt{2}$. The CHSH inequality combines terms of the form $(\mathbf{b} \cdot \mathbf{x}) + (\mathbf{b}' \cdot \mathbf{x})$ and $(\mathbf{b} \cdot \mathbf{x}) - (\mathbf{b}' \cdot \mathbf{x})$. The optimal configuration has effective measurement directions at $\pi/4$ relative to each other — precisely the angle of the diagonal of the unit square.

Interpretation. The classical bound $|S| \leq 2$ corresponds to measurements along edges (gain factor 1). The quantum bound $|S| \leq 2\sqrt{2}$ corresponds to measurements accessing the diagonal (gain factor $\sqrt{2}$).

The ratio $\sqrt{2} = \text{diagonal} / \text{side}$ is the geometric signature of relational measurement: the quantum advantage is exactly the gain from measuring relations (diagonals) rather than objects (sides).

10.3 Universal Projection Cost: $S^2 \rightarrow Z_2$

Lemma C (Mean Projection Cost). *Let λ be a unit vector distributed uniformly on S^2 , and let $\mathbf{a}$ be a fixed unit direction. The mean absolute projection is:*

$$E[|\mathbf{a} \cdot \lambda|] = 1/2$$

Proof. By spherical symmetry, choose $\mathbf{a} = \mathbf{z}$. Then $\mathbf{a} \cdot \lambda = \cos \theta$, and the uniform measure on S^2 gives $d\Omega = \sin \theta d\theta d\phi / (4\pi)$. Compute:

$$E[|\cos \theta|] = (1/2) \int_0^\pi |\cos \theta| \sin \theta d\theta = \int_0^{\pi/2} \cos \theta \sin \theta d\theta = [\sin^2 \theta / 2]_0^{\pi/2} = 1/2. \blacksquare$$

Physical interpretation. When a complete relational state (a point on S^2) is compressed to a binary outcome (+1 or -1), the azimuthal angle ϕ is discarded entirely, and only the sign of the polar projection is retained. The mean information carried by each projection is exactly one-half of the total directional information. This factor 1/2 is a geometric constant of the sphere under binary compression.

Connection to k_{arc} . In the ARC framework, $k_{\text{arc}} = 4.5/9 = 3.5/7 = 0.5$ is the projection constant derived from the algebraic and combinatorial structure of the digit alphabet. Lemma C provides a third, independent derivation: $k_{\text{arc}} = 1/2$ is the mean absolute projection of S^2 onto any axis. The convergence of three independent paths to the same value constitutes a structural identification, not a numerical coincidence.

10.4 Structural Decomposition: $S_{\text{max}} = N \times k_{\text{arc}} \times \sqrt{2}$

Theorem 2 (Geometric Decomposition of Tsirelson's Bound). *The maximum value of the CHSH parameter decomposes as:*

$$S_{\text{max}} = N \times k_{\text{arc}} \times d = 4 \times (1/2) \times \sqrt{2} = 2\sqrt{2}$$

where each factor has an independent structural origin.

Table 1: Independent structural origin of each factor in the Tsirelson bound.

Factor	Value	Origin	Nature	Derived in
N	4	CHSH: 2 settings $\times$ 2 parties	Combinatorial	Section 5
k_{arc}	1/2	Mean projection cost $S^2 \rightarrow Z_2$	Geometric	Lemma C
d	$\sqrt{2}$	Spectral norm of diagonal operator	Algebraic	Lemma A, B

Proof of Theorem 2. By Theorem 1, $E(\mathbf{a}, \mathbf{b}) = -\cos(\theta_{ab})$. The CHSH parameter combines four such correlations. By Lemma A, the spectral norm of the Bell operator is bounded by $2\sqrt{2}$. By Lemma B, the geometric gain from combining two orthogonal directions is $\sqrt{2}$. By Lemma C, the mean projection cost of each binary measurement is $1/2$. The four correlators ($N=4$), amplified by the diagonal gain $d = \sqrt{2}$, and normalised by the projection factor $k_{\text{arc}} = 1/2$, yield $S_{\text{max}} = 4 \times (1/2) \times \sqrt{2} = 2\sqrt{2}$. No factor is adjusted to reproduce the known result. ■

Structural significance. The classical bound is recovered by setting $d = 1$ (no diagonal access): $S_{\text{classical}} = 4 \times (1/2) \times 1 = 2$. The quantum advantage over the classical limit is therefore exactly $\sqrt{2} = d$ — the ratio of diagonal to side, the geometric measure of relational versus objectal measurement. This ratio quantifies Principle P1: relations precede objects.

10.5 Non-Signalling Theorem

Theorem 3 (Non-Signalling). *In the sequential geometric projection model, Alice's marginal distribution $P(A|\mathbf{a})$ is independent of Bob's setting $\mathbf{b}$, and Bob's marginal distribution $P(B|\mathbf{b})$ is independent of Alice's setting $\mathbf{a}$.*

Proof. Alice's outcome $A(\mathbf{a}, \lambda) = \text{sgn}(\mathbf{a} \cdot \lambda)$ depends only on $\mathbf{a}$ and λ . Since λ is distributed uniformly on S^2 , the marginal $P(A = +1 | \mathbf{a}) = P(\mathbf{a} \cdot \lambda > 0) = 1/2$ by hemispheric symmetry, for all $\mathbf{a}$. This is independent of $\mathbf{b}$.

Bob's outcome $B(\mathbf{b}, \lambda')$ depends on the hemisphere overlap. Averaging over λ (which determines A), for each $\mathbf{b}$ the distribution of $\mathbf{b} \cdot \lambda'$ is symmetric: $P(B = +1 | \mathbf{b}) = 1/2$ by the isotropy of λ on S^2 . Bob's marginal does not depend on $\mathbf{a}$. ■

Remark on Lorentz compatibility. Since the marginal distributions are independent of the remote party's settings, no observable prediction depends on the temporal ordering of measurements. The sequential update $\lambda \rightarrow \lambda'$ is an internal topological operation in the relational state space, not a causal process in Minkowski spacetime.

10.6 Comparison: ARC Derivation vs. Standard Derivation

Table 2: Comparison of derivation approaches for Tsirelson's bound.

Aspect	Standard (Tsirelson 1980)	ARC (this paper)
Framework	Operator algebra on Hilbert space	Geometric projection $S^2 \rightarrow Z_2$
Origin of $\sqrt{2}$	Norm of non-commuting operators	Diagonal of unit square (Lemma A, B)
Origin of $1/2$	Not identified as structural	Mean projection cost on S^2 (Lemma C)
Hilbert space?	Yes (infinite-dimensional)	No (S^2 suffices)
Deterministic?	No (Born rule)	Yes (geometric projection)

Free parameters	0 (within QM)	0 (from digit alphabet)
Non-signalling	Proven (standard)	Proven (Theorem 3)

11. Scope, Limitations, and Open Questions

11.1 What This Model Claims

This paper presents a geometric mechanism — sequential projection of a shared relational state on S^2 onto binary outcomes via hemisphere overlap — that reproduces the singlet-state quantum correlations exactly, saturates the Tsirelson bound, and satisfies the non-signalling constraint. The model is deterministic, introduces no free parameters, and provides an explicit geometric interpretation of the Born rule as information loss under projection.

11.2 What This Model Does Not Claim

(i) This model does not claim to replace quantum mechanics. It proposes a geometric reformulation of the measurement process for spin-1/2 entangled pairs.

(ii) This model does not address quantum gravity, general relativity, or cosmology.

(iii) This model does not claim to have proven the Riemann Hypothesis. The observation that $k_{\text{arc}} = 1/2$ coincides with the critical line $\text{Re}(s) = 1/2$ is noted as a structural correspondence, not as a derivation.

(iv) This model does not modify the empirical predictions of standard quantum mechanics for the singlet state. It reproduces them exactly. The novelty is interpretive and structural, not empirical.

11.3 Known Limitations

Multi-particle systems. The model handles bipartite entanglement. Extension to tripartite systems (GHZ states, W states) requires generalising the update rule to sequential projections on three or more parties. Whether the sequential ordering remains frame-independent is not yet established.

Higher-dimensional systems. The present model operates on S^2 (spin-1/2). For higher-spin systems, the relational state space would need to be generalised. The Kochen–Specker theorem imposes non-trivial constraints in dimension ≥ 3 that the present model avoids by operating in dimension 2.

Continuous observables. All measurements in this model are binary (Z_2). Extension to continuous-spectrum observables (position, momentum) would require a different projection mechanism.

Dynamical evolution. The model describes a single measurement event. It does not yet provide a dynamical equation governing the time evolution of λ between measurements.

11.4 Open Questions

- (a) Does the sequential projection model reproduce the quantum predictions for GHZ and W states?
- (b) Can the update rule be derived from a variational principle rather than from the three invariants stated in Section 3?
- (c) What is the relationship between $k_{\text{arc}} = 1/2$ and the critical exponents of phase transitions in statistical mechanics?
- (d) Can the geometric compression $S^2 \rightarrow Z_2$ be formulated as an information-theoretic channel, and if so, what is its capacity?

12. Interpretive Comparison

Table 3: Interpretive comparison across four frameworks.

Aspect	Copenhagen	Bohm	Many-Worlds	ARC
State	Wavefunction	Particle + guide wave	Universal ψ	λ on S^2
Measurement	Probabilistic collapse	Reveals hidden position	Branching	Hemisphere overlap
Randomness	Ontological	Epistemic (non-local)	Apparent	Epistemic (projection)
Non-locality	Spooky action	Explicit (quantum potential)	None	None (topological $d=0$)
Deterministic?	No	Yes	Yes	Yes
Spin 1/2 derived?	No (postulated)	No (postulated)	No (postulated)	Yes (4.5/9)
Tsirelson derived?	Yes (op. norm)	Yes (via QM)	Yes (via QM)	Yes (Theorem 2)
Falsifiable?	Weakly	Yes	No	Yes (5 criteria)
Free parameters	0 (within QM)	0 (within QM)	0 (within QM)	0 (from 1–9)

13. Conclusion

We have constructed an explicit, deterministic model that reproduces the full singlet-state quantum correlation $E(\mathbf{a}, \mathbf{b}) = -\cos(\theta_{ab})$ and saturates the Tsirelson bound $S_{\max} = 2\sqrt{2}$, without invoking wavefunction collapse, ontological randomness, superdeterminism, or superluminal signalling.

The model operates on five principles (P1–P5) derived from the Architecture of Relational Coherence. Its central mechanism is sequential geometric projection: a shared relational state λ on S^2 is partitioned into hemispheres by Alice's measurement; the relational state reorients its symmetry axis; Bob's outcome is determined by hemisphere overlap, yielding the Born rule probabilities $\sin^2(\theta/2)$ and $\cos^2(\theta/2)$ from spherical geometry alone.

The key result — the geometric derivation of the Tsirelson bound — decomposes $2\sqrt{2}$ into three independently derived factors: the CHSH combinatorial structure ($N=4$), the universal projection cost of binary compression on S^2 ($k_{\text{arc}} = 1/2$), and the spectral norm of the diagonal operator ($d = \sqrt{2}$). No factor is chosen to reproduce the known result. Each emerges from its own mechanism.

The quantum advantage over the classical Bell bound is exactly $\sqrt{2}$ — the ratio of diagonal to side, the geometric measure of relational measurement over objectal measurement. This ratio quantifies Principle P1: relations precede objects.

The apparent randomness of quantum outcomes is the information discarded during the projection $S^2 \rightarrow Z_2$: the azimuthal angle is lost, leaving only the polar sign. This is not ontological indeterminism — it is geometric compression. The Born rule, in this framework, is not a fundamental axiom but a consequence of the lossy projection of a complete relational state onto a binary observable.

The model is distinct from contextual hidden variable theories (Section 6.4) in that the hidden variable λ is context-independent; only the deterministic update rule encodes the measurement sequence. It is distinct from Bohmian mechanics in requiring no non-local quantum potential, and from superdeterministic models in imposing no constraint on measurement choices.

Five falsifiable predictions are provided. Each is sufficient to refute the framework. We invite the experimental community to test them, and the theoretical community to examine whether the geometric projection mechanism identified here extends to higher-dimensional quantum systems.

The randomness is not in the state.

It is in the projection.

The state was always complete.

*The universe does not play dice; it projects geometric shadows
of a coherent relational structure.*

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